The Problem of Unity and Multiplicity

A Structural Analogy between Arrow, Leibniz, and the All-at-Once Approach in physics

1.Introduction

The problem of the relation between unity and multiplicity—whether approached from unity toward multiplicity or from multiplicity toward unity—is one of philosophy's most enduring and fundamental problems. This essay explores a structural analogy between different manifestations of this problem across distinct theoretical frameworks.

Kenneth Arrow, awarded the Nobel Prize in Economic Sciences in 1972, demonstrates the impossibility of deriving a social ordering from unrestricted individual preferences while simultaneously satisfying the conditions of unanimity (Pareto), independence of irrelevant alternatives, and non-dictatorship. The philosophical move from this formal impossibility to the broader problem of deriving unity from plurality is my own interpretative step. It is on the basis of this interpretation that I develop the structural analogy between Arrow's impossibility theorem, Leibniz's metaphysics of pre-established harmony, and the All-at-Once approach in contemporary physics.

I begin with Arrow's theorem and use its formal structure as the starting point for developing the subsequent analogy. The aim is not to establish an equivalence between the theories under consideration, but to examine whether the structural perspective derived from Arrow can provide a productive framework for illuminating the different ways in which plurality, coherence, and unity are conceptualized. Such a comparison may, in turn, contribute to a clearer understanding of the respective assumptions, difficulties, and strengths of the theories examined.

2. Arrow's impossibility theorem

Arrow’s impossibility theorem [1] provides a formal expression of a fundamental problem concerning the relation between plurality and unity. Its original domain is social choice theory: given a plurality of individuals, each possessing an ordering of a set of alternatives, under what conditions can these individual orderings be transformed into a single, coherent social ordering?

In its abstract form, the problem can be represented as

$$(R_1, R_2,\ldots,R_n) \to R$$

where Ri denotes the preference ordering of individual i, and R the resulting social ordering. A *social welfare function* (SWF) is the rule f that performs this transformation. Arrow’s theorem establishes that no such function can simultaneously satisfy a set of apparently reasonable conditions. [1]

The significance of the theorem extends beyond the familiar observation that majority voting can generate cyclical outcomes. Arrow does not merely identify a defect of a particular voting procedure. Rather, he establishes a general impossibility concerning the aggregation of individual orderings into a collective ordering under specified conditions. In this sense, the theorem concerns the structural conditions under which a coherent unity can—or cannot—be derived from a plurality of independently ordered perspectives. [1]

2.1 The Conditions of Social Aggregation [1]

2.1.1. The first requirement is unrestricted domain (Condition U) [1]. The social welfare function must be capable of processing every logically possible combination of individual preference orderings. The admissible domain cannot therefore be restricted in advance to profiles exhibiting a particular degree of agreement or structural similarity. The individuals are, within the formal framework, free to order the alternatives in any logically permissible way.

This condition is important for the interpretation pursued here. Coherence cannot simply be presupposed at the level of the individual orderings. The problem posed by Arrow begins precisely with a plurality whose internal relations have not already been harmonized. If coherence is to emerge, it must be produced by the aggregation rule rather than introduced through a prior restriction of the plurality itself.

2.1.2 The second requirement is the Pareto principle (Condition P) [1]. If every individual strictly prefers an alternative *x* to an alternative *y*, the social ordering must likewise rank *x* above *y*:

$$\forall i \in N,\ xP_iy \Rightarrow xPy.$$

Thus, unanimous individual agreement must be preserved at the collective level. The aggregation procedure cannot reverse a comparison on which all individual orderings agree.

2.1.3. The third requirement is the independence of irrelevant alternatives (Condition I) [1]. The social ranking of two alternatives *x* and *y* must depend exclusively on the individuals' relative rankings of *x* and *y*. If their preferences concerning some third alternative zz change while their relative rankings of *x* and *y* remain unchanged, the social relation between *x* and *y* must remain unchanged as well. In other words, the collective comparison of a pair of alternatives is determined locally by the individual comparisons of that same pair. [1]

2.1.4. Finally, Condition D[1], non-dictatorship, requires that there be no individual whose strict preference between alternatives determines the corresponding social preference independently of the preferences of all other individuals. A dictator would be an individual ii such that, for every relevant pair *x*,*y*, [1]

xPiy ⇒xPy

irrespective of how the remaining individuals rank *x* and *y*.

The social welfare function is, moreover, required to yield a social ordering. Consequently, the resulting relation must possess the requisite consistency properties. In particular, transitivity requires that

xPy ◦ yPz ⇒ xPz

It is precisely the interaction between such global consistency and the conditions governing the aggregation of individual orderings that generates the impossibility result. [1]

2.2 The Impossibility Result

Sen states the General Possibility Theorem in its negative form:

<u>There is no SWF satisfying conditions U,P,I, and D.</u> [1]

Thus, unrestricted individual preference formation, the preservation of unanimous preference, independence of irrelevant alternatives, and non-dictatorship cannot all be maintained within a social welfare function yielding the required social ordering.

The theorem can therefore also be read from the opposite direction. If unrestricted domain, the Pareto principle, independence of irrelevant alternatives, and the requirements of a coherent social ordering are retained, the resulting aggregation rule must be dictatorial. Schematically,

<u>Unrestricted Plurality+Pareto+IIA+Collective Coherence ⇒ Dictatorship</u>

This formulation makes clear why Arrow's theorem is stronger than the Condorcet paradox. A Condorcet cycle demonstrates that one particular method—pairwise majority rule—can fail to generate a transitive social ordering.

Arrow establishes that the difficulty cannot simply be eliminated by replacing majority rule with some other aggregation procedure while retaining all the specified conditions.

2.3 Decisiveness and the Structure of the Proof

The central mechanism of the proof becomes particularly transparent through Sen's concepts of *almost decisive* and *decisive* sets. [1]

Let V be a subset of the individuals. V is *almost decisive* for x against y if x is socially preferred to y whenever every member of V prefers x to y and every individual outside V prefers y to x. Thus, V is capable of determining the social comparison even under maximal opposition from its complement. [1]

Sen's Lemma 3*a establishes the crucial expansion of this power: if a set of individuals is almost decisive for any ordered pair of alternatives, then, under Conditions U, P, and I, it is decisive for every ordered pair. The proof introduces a third alternative z and strategically varies its position in the individual preference orderings. Transitivity connects the resulting pairwise social comparisons, while independence ensures that changes involving z cannot alter a social comparison between x and y when the individuals' relative rankings of x and y have remained unchanged. In this manner, decisiveness with respect to one comparison propagates to other comparisons. [1]

This step reveals why the existence of at least three alternatives is structurally important. With only two alternatives, there is no third element through which pairwise relations can be connected by transitivity. Once a third alternative is available, however, apparently local decision-making power can be extended across the social ordering. [1]

The second major step concerns the size of a decisive set. The set of all individuals is decisive by virtue of the Pareto principle. One may therefore consider a smallest decisive subset V. Suppose that V contains more than one individual. Sen's proof divides this set into smaller parts and constructs suitable preference profiles over three alternatives. The resulting comparisons imply that one of these proper subsets must itself possess the relevant decisiveness. By Lemma 3*a, that subset would then be decisive generally. This contradicts the assumption that V was a smallest decisive set. [1]

The minimal decisive set must therefore contain exactly one individual:

$|V|=1$.

Since Lemma 3*a extends decisiveness from one ordered pair to every ordered pair, this individual is decisive throughout the social ordering. The individual is consequently a dictator.

The decisive point is not that one individual happens to acquire disproportionate influence. Rather, dictatorship emerges as a structural consequence of maintaining the other conditions simultaneously. [1]

2.4 From Social Choice to the Problem of Unity and Plurality

For the purposes of the present essay, Arrow's theorem will be treated not as a direct model for metaphysics, but as a formal structure from which a more general philosophical question can be extracted. Arrow begins with a plurality of individually coherent orderings,

$R_1, R_2, \ldots, R_n$,

and asks whether they can be transformed into one coherent ordering *R* while preserving specified forms of autonomy, independence, and agreement. His result demonstrates that these requirements impose mutually incompatible constraints upon the passage from individual to collective order.

The philosophical interpretation proposed here therefore concerns the conditions under which unity can emerge from plurality. It is essential, however, to distinguish this structural interpretation from Arrow's theorem itself. Individual preference orderings are not metaphysical substances, a social welfare function is not a theory of ontological composition, and dictatorship in Arrow's technical sense is not equivalent to metaphysical sovereignty. The analogy concerns the architecture of the problem rather than the identity of its elements.

Abstracted to this level, Arrow presents a configuration in which the preservation of certain properties of the plurality constrains the possible constitution of the whole. Most importantly, the theorem shows that coherence cannot simply be assumed to arise neutrally from unrestricted and independently structured individual orderings. Under Arrow's conditions, the demand for a single coherent collective ordering ultimately concentrates decisiveness in one element of the plurality.

In what follows, a distinction must be drawn between the Pareto condition of unanimity and what I shall call complete unanimity. The former requires the social ordering to preserve any pairwise preference shared by all individuals. Complete unanimity, by contrast, denotes the limiting case in which all individual preference orderings coincide: $R_1 = R_2 = \ldots = R_n$. In such a profile, plurality remains at the level of individual perspectives, while disagreement concerning the ordering itself disappears. It is this second, broader structural notion of unanimity—not the Pareto condition as such—that will become relevant to the subsequent comparison with Leibniz and the All-at-Once approach.

2.5. Transition to Leibniz

The next chapter turns to Leibniz's Monadology, which can be approached by posing a structurally related question at the metaphysical level: How can a multiplicity of autonomous substances constitute a single coherent world?

Leibniz offers a metaphysical conception of unity in which coordination precedes interaction. When read in structural relation to Arrow's theorem, this conception can be understood as achieving coherence not through interaction or aggregation, but through the prior establishment of a correspondence among autonomous substances—at the cost of relinquishing certain conditions that would otherwise govern the derivation of unity from multiplicity.

3. Leibniz' Monadology and metaphysics of pre-established harmony

In his Monadology, Leibniz constructs a philosophical system articulated in ninety propositions, thereby presenting a conceptual framework that challenged central assumptions of the Western philosophical tradition. The Monadology does not constitute a comprehensive exposition of Leibniz's metaphysics. Rather, it provides a conceptual formulation and a particular perspective on its fundamental notions, while subsequently exerting a significant influence on the interpretation of Leibniz's philosophy itself. [2]

Leibniz was also a mathematician. In 1684, he published his reflections on differential calculus in the *Acta Eruditorum*. His mathematical notation subsequently became established and remains, in its essential features, in use to this day.[3]

This mathematical background is relevant to the present argument. Leibniz's metaphysical framework should not be understood simply as the product of an inability to conceive of nature in mathematical terms. As one of the principal developers of differential and integral calculus, he was himself deeply involved in extending the mathematical means by which change and physical processes could be described. His recourse to metaphysics cannot simply be dismissed as a substitute for mathematical explanatory resources unavailable to him; rather, it can therefore be read as reflecting his conviction that mathematical description alone did not exhaust the question of the ultimate coherence and intelligibility of the world.

For Leibniz, reality is infinitely differentiated in its qualitative and conceptual content. It therefore admits of no complete identity of content between two distinct entities and is, in this sense, intrinsically individuated. Space and time cannot serve as principles of individuation. Leibniz emphasizes that two distinct subjects cannot possess exactly the same individual properties: every property of an individual is inseparably embedded in the concept of that individual and is therefore necessarily connected with all of its other properties. [4]

Leibniz further attributes a principle of completeness to the concept of the individual: through its complete concept, each individual is situated within the universal context of the world, insofar as its relations to all coexisting individuals and to the relevant states of affairs are implicitly contained within it. In this sense, the complete concept of an individual embeds its determinations within the total order of the world, thereby severely restricting the role that genuine independent causal contingency can play. [5]

By rejecting any real causal interaction between created substances, Leibniz is consequently driven, in order to account for their correspondence, towards a form of holism. Given his premises, a connection between individuals is possible only if they are understood as emanations from God.[6] All things, in all their realities—that is, in all their determinate contents—are continuously produced through the power of God and are therefore expressions of the whole, whose ultimate unity is constituted by God.[6]

Leibniz thus conceives reality explicitly in terms of both individuality and totality.

For the correspondence among these causally closed systems to be guaranteed, they must unfold according to a programme established by God once and for all in advance. Leibniz refers to this principle as pre-established harmony.[7]

Pre-established harmony therefore functions as a condition of stability of the world. [8]

3.1 Monadology

Several propositions[9] from the Monadology are particularly relevant to the present argument:

1 “The Monad, of which we shall speak here, is nothing but a simple substance which enters into compounds; simple, that is to say, without parts.”

11 “It follows from what has just been said that the natural changes of the Monads proceed from an internal principle, since an external cause cannot influence their interior.”

38 “Thus the ultimate reason of things must be in a necessary substance, in which the particular changes exist only eminently, as in their source; and this is what we call God.”

56 “Now this connection or accommodation of all created things to each other, and of each to all the others, brings it about that each simple substance has relations which express all the others, and consequently is a perpetual living mirror of the universe.”

78 “These principles have enabled me to explain in a natural way the union, or rather the correspondence, between the soul and the organic body. The soul follows its own laws, and the body likewise follows its own laws; and they correspond by virtue of the pre-established harmony between all substances, since they are all expressions of one and the same universe.”

3.2. conclusions

From these and other propositions of the Monadology, as well as from their subsequent interpretation throughout the history of philosophy, it becomes apparent that Leibniz addresses the problem of unity without interaction by replacing inter-substantial causation with a pre-established correspondence among autonomous perspectives.

In structural terms, Leibniz circumvents the problem of aggregation by introducing a principle of pre-established agreement. This may be compared not simply with Arrow's Pareto condition, but with the limiting case of complete unanimity introduced above: plurality persists, while the possibility of fundamental incompatibility among the individual perspectives is excluded in advance.

Rather than deriving unity from an aggregation or interaction of independently constituted substances, he guarantees their conformity through a prior order within which each substance expresses the same universe from its own perspective.

Leibniz thereby arrives at what may be characterized, with appropriate qualifications, as a form of metaphysical determinism. The complete or perfect concept of an individual substance encompasses its determinations across the entirety of its temporal existence, including its past, present, and future states (Liske, p. 108). God, in turn, functions as the necessary ground that holds together a rationalism founded upon the principle of sufficient reason.

Whereas Arrow, at the level of social choice theory, demonstrates that unity cannot be obtained through an unrestricted aggregation procedure while simultaneously satisfying all four conditions, Leibniz establishes pre-established harmony—and, by analogy, a form of unanimity—as a foundational principle of his metaphysics. At the same time, by positing God as the necessary creator and guarantor of this pre-established harmony, Leibniz arrives, in a manner that can be read as violating Arrow's non-dictatorship condition, at a coherent—although not necessarily universally convincing—account of how unity or totality can emerge from multiplicity.

3.3 transition to quantum physics

Here again, the purpose is not to establish an identity between the respective frameworks, but to identify a structural analogy and to examine the implications that can be derived from it.

A first structural parallel between Leibniz's Monadology and Quantum Physics identified by Ivancheva (2021)[10] concerns the relation between individuality and continuous transformation. Leibniz conceives monads as simple, indivisible substances—"true atoms of nature"—each qualitatively distinct from every other and without "windows" through which external influence could enter. At the same time, however, every monad is subject to continuous internal change, governed by an intrinsic principle rather than by external causation. Ivancheva interprets this conjunction of discrete individuality and continuous transformation as conceptually analogous to the wave–particle duality of quantum objects: elementary particles exhibit corpuscular properties while also possessing wave-like characteristics that resist description in exclusively classical terms. The analogy is therefore not one of physical identity between monads and quantum particles, but of structure: in both cases, what appears as an individual and discrete entity cannot be exhaustively characterized through the classical concept of an isolated, static particle. [10]

A second, and more consequential, parallel concerns the relation between local autonomy and global interconnectedness. Leibniz's monads do not causally interact, yet each expresses the universe from its own perspective; the state of an individual substance can ultimately be understood only in relation to the entire order to which it belongs. Ivancheva (2021) [10] relates this conception to quantum entanglement and non-locality, where the states of distinct entities may cease to be independently specifiable and instead form a unified state whose correlations cannot be reduced to the properties of spatially separated components. She emphasizes the striking affinity between this non-separability and Leibniz's claim that interconnection extends across arbitrary distances and that each constituent of reality bears a relation to the whole. The structural principle common to both pictures is thus that separateness at the level of individual entities does not entail independence at the level of the total state: plurality and unity coexist, while the whole cannot be reconstructed simply by treating its constituents as mutually independent elements. [10]

Finally, Ivancheva[10] extends this relation between part and whole through the analogy with holographic and fractal conceptions of the universe. The defining feature of a holographic structure relevant here is that information concerning the whole is, in some sense, contained within each of its parts; Ivancheva compares this explicitly with Leibniz's conception of every monad as a representation or "living mirror" of the entire universe. Each monad remains distinct and limited by the particular clarity of its perceptions, yet simultaneously expresses the same totality from its unique perspective. The resulting structure is neither a reduction of the whole to its parts nor a dissolution of the parts into an undifferentiated whole. Rather, it is a model in which multiplicity is internally related to totality: each element retains its individuality while its intelligibility depends upon, and reflects, the global order. [10]

Across Ivancheva's comparisons—from wave–particle duality through non-locality and entanglement to holographic structure—the recurrent structural motif is therefore the same: modern quantum physics and Leibnizian monadology both challenge a purely separative ontology in which the whole is nothing more than the externally assembled sum of independently constituted parts.

4.1. The All-At-Once-Approach

The All-at-Once Approach, proposed by Wharton (2015) [11], can be understood as a challenge to a largely implicit assumption about the explanatory structure of fundamental physics. What he calls the Newtonian Schema conceives the universe in analogy with a computational process: a physical state at one time functions as an input, dynamical laws transform this state, and subsequent states emerge as outputs. Although the mathematical form of physical theories has changed profoundly since Newton, Wharton argues that this basic explanatory architecture continues to dominate the interpretation of modern physics. The essential assumption is therefore not Newtonian mechanics as such, but a particular conception of temporal explanation:

initial state - dynamical laws - later state.

On this picture, the past possesses explanatory priority over the future: given sufficiently complete initial information and the appropriate laws, later physical states are to be derived from earlier ones. Wharton's central contention is that the usefulness of this procedure for calculating physical outcomes does not establish that the universe itself must instantiate the same computational architecture. [11]

The alternative is, what Wharton (2015) [11] refers to, the Lagrangian Schema. Its paradigmatic precursor is Fermat's principle of least time. A ray of light travelling between two points need not be represented as proceeding through a succession of locally computed states; rather, the relevant physical trajectory is selected by considering the path as a whole. More generally, Lagrangian mechanics imposes constraints upon the boundaries of a spacetime region and employs a global quantity—the action—to determine which of the possible histories satisfies the relevant condition. Crucially, the boundary conditions may include both initial and final constraints. The fundamental explanatory object is therefore no longer an instantaneous state that generates its successor, but an entire spacetime history subject to global constraints:

boundary constraints + global action principle → admissible four-dimensional history.

In this framework, the intermediate events are not successively computed from an initial state. They form parts of a solution defined over the spacetime region as a whole. This is the basis of Wharton's characterization of the universe as a global four-dimensional problem solved "all at once." [11]

This distinction becomes especially significant in quantum physics. Wharton[11] argues that several difficulties appearing profound within the Newtonian Schema acquire a different character once the demand for purely initial-state dynamics is abandoned. The uncertainty principle no longer threatens the completeness of an initial state, because the Lagrangian Schema does not require complete initial information in the first place. Likewise, the measurement problem is reframed when a final measurement outcome can enter the description as a boundary constraint rather than having to emerge as the dynamically generated output of a prior quantum state. Past and future constraints thereby acquire a formally more symmetrical status. [11]

Quantum entanglement is particularly important for the all-at-once conception. Within a Newtonian framework, correlations between distant systems seem to demand an account of how the state of one system is dynamically related to that of another. Wharton instead proposes treating the correlated events as components of the same globally constrained spacetime solution. The apparently "spooky" relation between entangled particles can then be understood as a joint correlation arising because both contribute to the same global action. The explanatory burden is transferred from a mechanism connecting separated events to a constraint applying to the complete configuration. [11]

The resulting difference between the two schemas is therefore deeper than a choice between alternative mathematical techniques. The Newtonian Schema is fundamentally dynamical: three-dimensional states are related through laws of temporal evolution. The Lagrangian Schema can instead be formulated in terms of four-dimensional histories satisfying global restrictions. Wharton eventually expresses the contrast in particularly strong terms: such theories can be conceived as "all kinematics, no dynamics." Accordingly, in the relevant Lagrangian models there need be "no dynamics, local, nonlocal, or otherwise." His proposal is thus not primarily to discover a new causal mechanism capable of producing quantum correlations, but to question whether fundamental physics must possess this dynamical explanatory form at all. [11]

4.2. Interpretation within the Present Argument

The following step goes beyond Wharton's own terminology and constitutes the interpretation proposed in this essay. I suggest describing the contrast between the two schemas as a distinction between vertical and horizontal modes of unification. A vertical structure orders events through temporal dependence:

$$E_1 \rightarrow E_2 \rightarrow E_3 \rightarrow \dots,$$

where the state of the system at one moment causally or dynamically determines its state at a subsequent moment. The unity of the history is consequently produced through a chain of temporal determination. The all-at-once framework can instead be interpreted as horizontal:

$$E_1 - E_2 - E_3 - \dots - E_n \subset H,$$

where the individual events belong to a single globally constrained history *H*. Their coherence need not be produced sequentially; it consists in their joint compatibility with the same global solution. Wharton himself explicitly rejects the demand to identify which past event "causes" a future boundary constraint, arguing that such a question already presupposes the Newtonian Schema.

Within the conceptual framework of this essay, this global compatibility may be described—carefully and analogically—as a form of horizontal unanimity. I use it to designate a structural requirement according to which a plurality of locally distinguishable events must be mutually compatible as constituents of one global system. Horizontal unanimity should therefore not be understood as equivalent to Arrow's Pareto condition, but as a structural analogue of complete unanimity: the constituent events are not identical, but their admissibility requires their joint compatibility within the same global history. The problem of multiplicity and unity is thus not solved by introducing an additional element that acts upon the plurality from a superior level. Unity is imposed through the consistency conditions of the whole itself.

The contrast with Leibniz can then be formulated more precisely. Leibniz likewise confronts the possibility of coherence among entities that cannot explain their coordination through ordinary interaction. Yet the pre-established harmony ultimately introduces what, in the terminology proposed here, may be called a vertical metaphysical level: God grounds and guarantees the correspondence among the monads. At the same time, he introduces a horizontal dimension of metaphysical order, insofar as each individual monad stands in harmonious correspondence with the universe as a whole. In the all-at-once approach, by contrast, no corresponding external coordinating instance is required. If global consistency is taken as fundamental, there is no further need—within this explanatory structure—to postulate God as the cause or guarantor of that consistency. This conclusion is not advanced by Wharton himself; it follows only from the comparative interpretation developed in the present essay.

This apparent advantage, however, reveals what I propose to regard as the approach's most fundamental philosophical vulnerability. The absence of a vertical coordinating principle is achieved precisely by relinquishing the demand for a fundamental dynamical-causal explanation. Wharton relocates the explanatory stopping point from dynamical causation to global constraint. In his own terms, there may simply be no fundamental dynamics "local, nonlocal, or otherwise." My critical interpretation is therefore that the strength and weakness of the all-at-once approach coincide at exactly the same point: it can reconcile multiplicity with global unity without introducing a transcendent coordinating principle because it replaces vertical causal determination with horizontal global consistency; yet precisely for that reason, it cannot provide a causal account of the unity it describes. Whether this constitutes an explanatory liberation or an explanatory deficit depends ultimately upon whether causality is regarded as a necessary feature of fundamental explanation.

4.3 The All-at-Once Approach and Arrow

The connection to Arrow's impossibility theorem emerges once both frameworks are considered at a sufficiently abstract structural level. Arrow begins with a plurality of individually ordered perspectives and asks under what conditions they can be transformed into a single coherent collective ordering. His theorem demonstrates that, given unrestricted individual orderings, Pareto unanimity, independence of irrelevant alternatives, and the requirement of collective rationality, a non-dictatorial aggregation is impossible. The decisive structural tension therefore lies between the independence of the constituent orderings and the coherence required of the resulting whole. If the relevant conditions are retained, global consistency ultimately requires the concentration of decisiveness in a single individual. In the terminology developed in this essay, Arrow may consequently be read as exposing a limit to the possibility of constructing horizontal unity out of an unrestricted plurality while simultaneously preserving specified forms of local independence.

Wharton's all-at-once framework can be interpreted as responding to a structurally related problem by radically changing the architecture through which global coherence is obtained. Rather than beginning with independently specified local states and asking how one state dynamically generates another, the Lagrangian Schema treats the complete four-dimensional history as the fundamental object of description. Initial and final boundary conditions jointly constrain the admissible history, while the individual events within that history must satisfy a single global constraint. Quantum correlations therefore need not be generated through a dynamical connection between independently constituted elements; in Wharton's treatment of entanglement, the correlations between particles can instead be understood as joint correlations arising because both contribute to the same global action. The whole is consequently not constructed after its parts have been independently determined; the admissibility of the parts is itself constrained by their membership in the whole.

This suggests a particularly revealing contrast with Arrow. Arrow preserves unrestricted plurality and local independence and thereby encounters an impossibility of non-hierarchical global aggregation; Wharton obtains global coherence by abandoning the corresponding picture of independently determined states altogether. This is not a solution to Arrow's theorem in any mathematical sense, since the objects and axioms of social choice theory have no direct equivalents in Wharton's physics. It is, rather, a structural analogy. In the terminology proposed here, the all-at-once approach replaces aggregation with global constraint.

The direction of explanation is therefore fundamentally different. In Arrow, the individual orderings are given and the problem is to derive the whole from them; in the all-at-once framework, the global history constrains what can count as an admissible configuration of its constituent events in the first place. Wharton's characterization of the Lagrangian approach as a four-dimensional global problem, and ultimately as a framework with no fundamental dynamics, provides the physical basis for this interpretation.

This comparison also clarifies the role of the "vertical" dimension in the three frameworks considered here. In Arrow, the impossibility result leads, under the specified conditions, to a privileged individual whose preferences determine the collective ordering: the dictator. In Leibniz, the coherence of the plurality of windowless monads is guaranteed by a metaphysically prior principle, ultimately grounded in God and the pre-established harmony. Wharton, by contrast, introduces no analogous superior coordinating agent. On the interpretation advanced here, the all-at-once approach seeks coherence entirely at the horizontal level: the plurality is unified through global compatibility rather than through a causal or metaphysical agency standing above it. This is possible precisely because Wharton rejects the Newtonian demand that the history of the universe be explained through dynamical succession from past to future; in the Lagrangian Schema, there may be "no dynamics, local, nonlocal, or otherwise."

The explanatory gain of the all-at-once approach therefore comes at a considerable metaphysical price. By replacing dynamical evolution with global constraint, Wharton eliminates the need to explain how a plurality of events is successively coordinated through causal interaction. In the terminology proposed here, the vertical dimension of explanation is abandoned in favour of horizontal unanimity: each event must be compatible with one and the same globally constrained four-dimensional history. Yet this move does not explain how the universe proceeds from E_1 to E_2; more radically, it denies that such a transition requires a fundamental causal account at all. Nor does the framework provide a further causal ground for why the plurality of events constitutes precisely this globally consistent history. In contrast to Leibniz, no transcendent guarantor of harmony—and hence no God as first ground—is required. But the removal of this metaphysical postulate is purchased by an equally radical renunciation: global consistency replaces causal explanation rather than completing it. The central question is therefore whether the elimination of the demand for causality constitutes a genuine solution to the problem of unity and multiplicity, or whether it merely relocates the explanatory burden from causal coordination to an unexplained global consistency.

5. Conclusion

The resulting threefold comparison can thus be stated with some precision. Arrow demonstrates – within social choice theory –the impossibility of obtaining a coherent social ordering from unrestricted plurality while simultaneously preserving all of the specified conditions; Leibniz secures unity by grounding the plurality in a prior metaphysical coordination; the All-At-Once-Approach secures global consistency by ceasing to treat the parts as independently determined inputs from which the whole must subsequently be generated. In this sense, Leibniz and the All-At-Once-Approach represent two fundamentally different responses to the structural problem illuminated by Arrow.

Neither Leibniz nor Wharton provides an alternative aggregation rule capable of overcoming Arrow's impossibility. Rather, both alter the structural conditions under which an Arrow-like aggregation problem can arise in the first place.

Leibniz retains a vertical principle of explanation and locates it in God; Wharton eliminates the need for such a vertical principle by replacing dynamical production with global constraint. The price of the latter move, according to the interpretation proposed in this essay, is equally fundamental: where Leibniz can answer the question of why the plurality is harmonized by referring to its metaphysical ground, the All-At-Once-framework deliberately ceases to demand a causal explanation of global consistency. The all-at-once approach thereby avoids the need for a "dictator" or external coordinator, but only by transforming the very relation between plurality and whole that makes an Arrow-like aggregation problem arise in the first place, while nevertheless retaining a structural analogue of complete unanimity in the form of horizontal compatibility: all constituent events must be compatible with one and the same globally constrained history.

The purpose of this essay is not to propose a solution to the fundamental problem of unity and multiplicity, but to investigate how this problem manifests itself across distinct theoretical frameworks. By identifying their structural similarities and differences, the analysis may illuminate the respective strengths and limitations of these approaches and thereby contribute to their critical evaluation within a broader scientific and philosophical context.

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